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Theoretical investigation and numerical simulation of radiative outflows around compact objects

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Abstract. Winds and Jets form in active galaxies and in binary systems which are known to harbour compact objects at the centre. Winds are likely to originate from centrifugal barriers in advective accretion flows, but the acceleration of the flow is a puzzle. Matter likely to start subsonically from a disk must be accelerated very close to the black hole in order to reach a velocity comparable to the velocity of light, which is actually observed. But the terminal velocity achieved by jets are still an enigma. We try to answer some of these questions by studying critical point behaviour of the outflow in presence of radiative acceleration. We show that the momentum deposition term changes the character of the solution drastically depending on the magnitude and the exact location of the deposition. We have been able to accelerate the matter from very close to the compact object to infinity, particularly interesting is the case, where we see radiative momentum deposition force (hereafter RAMOD) actually drive away bound matter as winds. We also study the time evolution of the outflows by numerical simulation, and find these new solutions to be stable.

Keywords : Jets, outflows, radiative transfer, black holes

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1. Introduction

Jets and outflows are observed in Galactic and Extra-galactic compact systems. Although jets are generally associated with compact objects like black holes and neutron stars [1], but the terminal velocity achieved by these jets are observed to vary remarkably from case to case. In some cases it is observed to be comparable to the velocity of light (0.99c), in others the terminal velocity is observed to be much lower around 0.26c (e.g. for SS433, [2]). Thus the actual process of acceleration of jets resulting in such varying terminal velocities, is still an enigma.

There are various models which try to explain acceleration of jets and outflows. Blandford & Payne [3] studied self-similar outflows from magnetized accretion disks and the attributed the acceleration to be due to centrifugal force. Contopoulos & Lovelace [4] suggested electrodynamical acceleration of jets and outflows. Meier [5] found out several regimes of dynamical wind structure in presence of super-critical accretion. In the stellar wind front, several works pioneered by Castor and his collaborators (e.g., [6]) found nature of the radiation driven winds. However, so far, no work has been done to find the variation of solution topologies due to photon 0 deposition very close to the black hole or a neutron star. Matter emerging with very low velocity is continuously bombarded with higher energy photons, and hence apart from the usual thermal acceleration, deposited momenta by the hot photons can accelerate matter. If RAMOD is large enough it can even drive away bound matter as wind.

In the next section we present equations governing the motion and discuss the method of solution. In §3.1 we present results of our theoretical investigation. In §3.2 we discuss about the simulation and also the results it produces. Finally in §4 we present our concluding remark.

The units of length, mass, time and velocity are $\frac{2GM_{BH}}{c^2}$, M_{BH}, $\frac{2GM_{BH}}{c^3}$, c, respectively, where M_{BH} = mass of the black hole, c = velocity (not to be confused with suffix c) of light, G = gravitational constant. Henceforth every variable is expressed in geometrical units explained above.

2. Method

We assume that the outflow is coming out radially, i.e it out has no angular momentum. The flow boundary is conical. with the compact object sitting on the origin of the spherical coordinate system. The outflow is also considered to be inviscid. We consider the gravitational potential to be the so called 'Paczyński-Wiita' potential (Paczyński & Wiita, [7]), instead of the usual Newtonian one. This little sleight of 'hand' in fact enable us to by pass the usual general relativistic treatment, while loosing very little in terms of physics, as this potential can mimic the Schwarzschild geometry (the error is within a few percentage). Presently, we are not trying to connect the accretion disc and the conical outflow.

The equations governing the steady state motion are :

The Momentum Balance Equation

$$v\frac{dv}{dr}+\frac{1}{\rho}\frac{dP}{dr}+\frac{1}{2(r-1)^2}-\mathcal{D}_0\exp[-(\frac{r}{r_c}-0.5)^2]=0 \tag{1}$$

The Baryon Number Conservation Equation (apart from a geometrical factor)

$$M=\rho v r^2(1-\cos\theta_0) \tag{2}$$

The Entropy Generation Equation

$$T\frac{ds}{dr} = \mathcal{D}_0\exp[-(\frac{r}{r_c} - 0.5)^2] \tag{3}$$

We have to solve these three equations with the help of ideal gas equation

$$P_{gas} = \frac{\rho k_B T}{\mu m_p} \tag{4}$$

The expression of adiabatic sound speed is $a^2 = \frac{\gamma P}{\rho}$. The ratio β of gas pressure P_{gas} and total pressure P ($= P_{gas} + P_{rad}$; P_{rad} = radiation pressure) is constant, $\beta = \frac{P_{gas}}{P}$.

In the above equations and expressions,

$\dot{M}$ = mass outflow rate (as this is a steady state motion, $\dot{M}$ =constant), v = radial velocity, ρ = matter density, a = sound velocity, r = radial distance, s = specific entropy of the flow, r_c = sonic point or critical point of the flow (defined later), T = temperature of the flow (the electron and proton temperature are assumed to be same), γ = ratio of specific heats , $\frac{c_P}{c_V}$, (here $\gamma = \frac{4}{3}$), θ_0 = semi vertical angle (assumed to be constant), μ = number of electrons per nucleon, m_p = mass of the proton, k_B = Boltzmann constant.

The two forces involved here are the gravitational force ,i.e.,

$$F_{grav} = \frac{1}{2(r-1)^2}, \tag{5}$$

and RAMOD,

$$\mathcal{D}_r = \mathcal{D}_0\exp[-(\frac{r}{r_c} - 0.5)^2]. \tag{6}$$

It is to be remembered that both these forces are radial. Hence essentially this is modified Bondi outflow [8]. In equation (6), $\mathcal{D}_0$ is actually maximum $\mathcal{D}_r$, i.e., at $r = 0.5r_c$, $\mathcal{D}_{r(max)} = \mathcal{D}_0$. The expression of $\mathcal{D}_0$ is

$$\mathcal{D}_0 = \frac{L\sigma_T}{4\pi[(0.5r_c)^2 + (10)^2]cm_p}$$

where $L = \eta\dot{M}_{acc}c^2$ = Luminosity of the accretion disk, $\dot{M}_{acc} = f\dot{M}_{EDD}$; f is a non-dimensional accretion rate, $\dot{M}_{acc}$ = true accretion rate, $\dot{M}_{EDD}$ = Eddington rate, η = efficiency of conversion of rest mass energy to radiation, in Schwarzschild case the highest efficiency is $\eta = 0.06$, σ_T = Thomson scattering cross section.

We have assumed, an annular ring of radius $10r_g$ ($r_g = \frac{2GM_{BH}}{c^2}$) on the accretion disk, as the source of the radiation which deposits momentum on the outflow. Our assumption, though is a simplification, but is based on the fact that, 'hard X-rays' generally come out from around $10r_g$ of the accretion disk,

as is found out by Chakrabarti & Titarchuk [9].
Now from the expression of RAMOD we can see that it is maximum at a radial distance half that of the critical point, and falls off about that point in a Gaussian nature.

2.1. *Sonic point analysis*

From equations (1), (2), (3), (4) and (5), we find the expression for $\frac{dv}{dr}$ and $\frac{da}{dr}$. Then following the so called *Sonic point Analysis* [10], we find *sonic point condition* or *critical point condition.*

$$v_c^2 = a_c^2, \tag{7}$$

and

$$a_c^2 = \frac{r_c^2}{4(r_c - 1)^2} + \frac{r_c(\gamma - 2)\mathcal{D}_0 \exp[-0.25]}{2}. \tag{8}$$

The radial distance at which sonic point conditions are satisfied is called the *critical point* or *sonic point*, denoted as r_c. Now the gradients of radial velocity and sonic velocity, $\frac{dv}{dr}$ and $\frac{da}{dr}$ respectively, at r_c can be found out by using L'Hospital rule. We integrate the expressions of $\frac{dv}{dr}$ and $\frac{da}{dr}$, from r_c up to 1.5 Schwarzschild radius, and then once again from r_c outwards, we get the complete analytic solution.

3. Results

3.1. *Theoretical result*

Observation tells us that the terminal velocity of galactic and extra galactic jets and outflows are generally comparable to the velocity of light. Present theoretical knowledge predicts, jets and outflows of compact objects are formed very close to the compact object with very small subsonic velocity. Therefore in between the initial and terminal states, outflows must pass through one or more sonic points. We in our investigation, on one hand, have not found more than one stable 'X- type' sonic point, which resembles a typical Bondi outflow type solution. On the other, from equation (8) we see, if RAMOD is large enough, then $a_c^2 \leq 0$, because $(\gamma - 2) < 0$, which is clearly unphysical. This implies there exist a cut off for r_c's beyond which a physical solution doesn't exist. Once the physically permissible domain is known from equation (8), we can then integrate and find our solutions. Figure 1(a) shows the comparison of the solutions of radial velocity v for RAMOD corresponding to $15\dot{M}_{EDD}$ rate of accretion and that due to Bondi outflow respectively, starting with same initial velocity of about 0.013c. We see the terminal velocity is nearly twice than that of the Bondi case. Hence we see that the radiation from the accretion disc can accelerate the outgoing matter twice more, than purely thermally driven winds. In fact depending upon the magnitude (which depends on the accretion rate of the disc), and the position of the momentum deposition, we found, the terminal velocity of the outflows can reach up to 50% of the velocity of light.

RAMOD not only accelerates outflowing matter, but can also drive out matter with negative specific energy. Figure 1(b) shows the variation of specific energy E with $log_{10}r$, RAMOD corresponds to $15\dot{M}_{EDD}$ rate of accretion. Clearly we see that matter with negative energy is driven out with positive energy. As RAMOD is Gaussian in nature, it ceases to act beyond its half-width, i.e beyond the half-width the outflow behaves like a Bondi outflow. Saturation of the specific energy to a constant value manifests this fact.

3.2. *Numerical Simulation*

The analytical results are compared with that of numerical simulation. Numerical simulation results are obtained by using a novel technique called 'Smooth Particle Hydrodynamics' or abbreviated as SPH. SPH code is essentially used in astrophysics to simulate shock waves in accretion flows onto compact objects. SPH is a Lagrangean method, and the complexities which arise while using Lagrangean method in two or three spatial dimensions is overcome by computing spatial derivatives through interpolation of neighbouring points. Hence SPH is also an 'interpolation method'. Interpolation points are known as 'particles', which are allowed to evolve in time under the guidance of usual fluid dynamical laws. For our purpose, we have put the angular momentum, viscous force equal to zero, and have introduced RAMOD, to make the code workable for one dimensional radial outflow, which is accelerated by thermal pressure and RAMOD. The principle idea of this simulation is, we inject particles each at regular interval of time and see the behaviour of these particles as it evolves in time under the influence of usual fluid dynamical laws. Initially the transient state dominates but as it reaches steady state we compare these solution with the theoretical ones. Figure 1(c) shows the comparison of the variation of Mach number M with $log_{10}r$, between numerical and theoretical results. The solid line represents the numerical simulation result, and the broken line represents the theoretical result. In both the case RAMOD corresponds to $10\dot{M}_{EDD}$ rate of accretion. We have incorporated r_c dynamically. Figure 1(d) shows the calculated r_c with time at steady state. We see that even at steady state it oscillates. Matter coming out with subsonic velocity has to overcome very strong gravity, but thermal pressure and RAMOD at the same time pushes it outward. Under these two contradictory forces matter tends to pile up, resulting in increased pressure. This then shoves matter out with high velocity, but as the outgoing matter has to follow the same dynamical laws as the theoretical result, it comes down towards the theoretical result, resulting in an oscillation. Thus we find that Mach number M computed using SPH code initially oscillates, as is vindicated by Fig. 1(c). The same information of oscillation is propagated while calculating r_c dynamically, as is shown in Fig. 1(d).

4. Conclusion

The model we have followed is based on some simplistic assumption. For example we haven't considered angular momentum of the outflow. Neither have we considered viscosity of the outflow. We have calculated RAMOD from the gravitational potential release of accreting matter, and not from the spectrum directly. We haven't even considered the cooling terms, like Bremsstrahlung, synchrotron radiation, inverse Comptonization. Still our analysis of radiative outflow gives a fair estimation of the role RAMOD plays in changing the solution topology of the outflow. Here we precisely present our concluding remarks, basing on the results we obtained.
(1)RAMOD working against gravity helps in accelerating the flow, the extent of which depends on the magnitude of RAMOD, and the position of the sonic point. The terminal velocity achieved is twice or thrice more than that achieved for Bondi type outflows. (2)RAMOD working against the gravity, changes the topology of the outflow (3)RAMOD increases the energy of the outflow, which increases with higher value of RAMOD. (4)RAMOD acting on the outflow limits the parameter space
(5)due to the action of RAMOD, matter with negative energy can also be driven off as outflows

4.1. *Further Improvement*

Throughout this work we have considered a simplistic situation, where the outflow has no angular momentum, and does not include viscosity. Furthermore, we didn't consider any cooling term, though we know a lot cooling process like Bremsstrahlung, inverse Comptonization etc actually do occur. Furthermore the radiation which we consider to originate only from a annular ring of $10r_g$ on the accretion disk is an over-simplification. We are considering the radiative contribution from the rest of the disk, angular momentum of the outflow, other cooling processes and viscosity in future.

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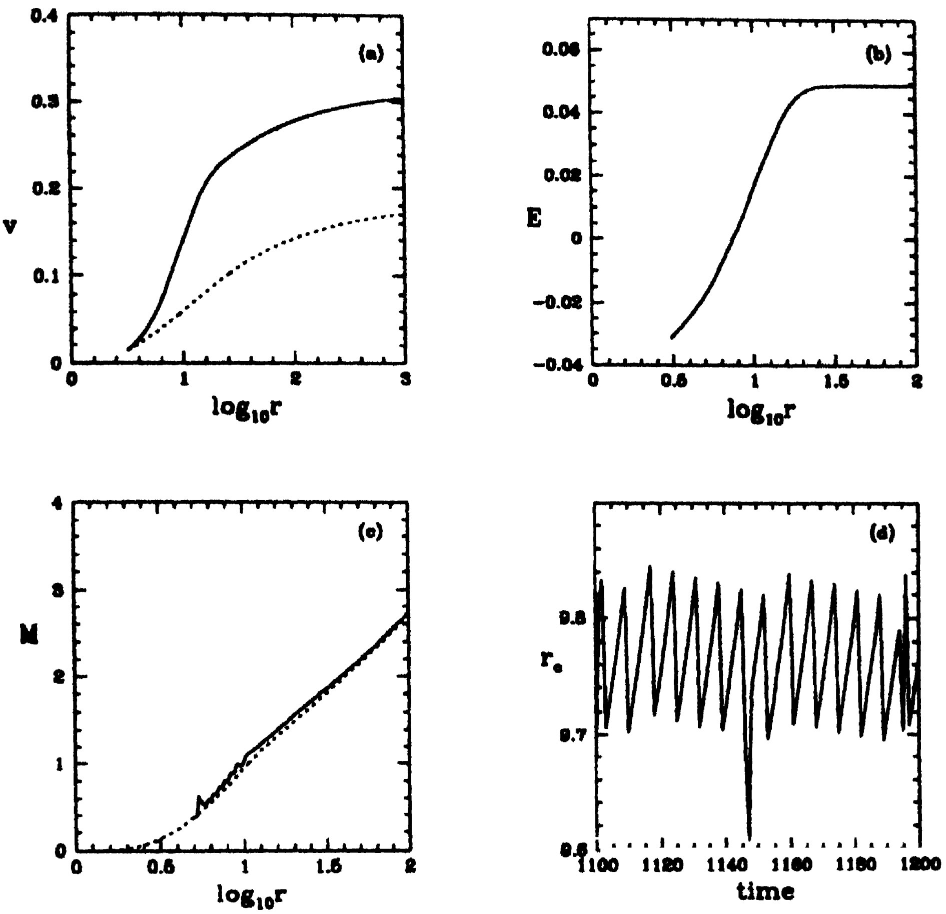

Figure 1a. Comparison of the velocity variation with $log_{10}r$. The solid curve presents the solution, acted on by RAMOD corresponding to $\dot{M}_{acc} = 15\dot{M}_{EDD}$. $r_c = 10.19r_g$. Dashed-curve presents Bondi Outflow solution. $r_c = 25.86r_g$. Initial position and velocity for both the case are $3.19r_g$ and 0.013c, respectively.

Figure 1b. Variation of specific energy with $log_{10}r$. RAMOD corresponds to $\dot{M}_{acc} = 15\dot{M}_{EDD}$. $r_c = 10.19r_g$. Initial energy E = −0.03127 (in units of Mc^2) at $r = 3.068r_g$.

Figure 1c. Comparison of Mach number variation with $log_{10}r$. Solid line corresponds to numerical simulation result. Dashed line corresponds to theoretical result. RAMOD corresponds to $\dot{M}_{acc} = 10\dot{M}_{EDD}$. Initial point, initial velocity, initial sound velocity are $5.054r_g$, $0.09668c$, $0.2550c$.

Figure 1d. Variation of sonic point with time, calculated dynamically by numerical simulation. RAMOD corresponds to $\dot{M}_{acc} = 10\dot{M}_{EDD}$.

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